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Section XII

Prestressed Brickwork

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Prestressed Brickwork

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INTRODUCTION

Brick Linings—A General Discussion, and the Problem of Tensile Stresses

The installation of a brick lining in a process vessel is a common design technique for protecting the vessel jacket from corrosive environments. A variety of highly chemically resistant bricks and mortars are available.

Since brick and masonry do not resist tensile stresses very well, the designer must be careful to ensure that tensile stresses are avoided over the entire range of operating and shutdown conditions. If a vessel will be operating at elevated temperatures or pressures, tensile stresses in the brickwork will be encountered, unless special design techniques are used. This is due primarily to the fact that the coefficient of thermal expansion of brick is typically half that of the surrounding steel jacket. Thus at elevated temperatures the steel jacket will try to expand away from the brickwork, causing the brickwork to be under tension. These tensile stresses will cause cracks in the brickwork, weakening the chemically protective barrier it affords, and also will cause the brickwork to pull away from the steel shell.

A Solution—Prestressing

These tensile stresses can be avoided by subjecting the vessel to a prestressing cure, which induces an artificial compression in the brick lining and a tension in the steel shell. When properly designed, the brick lining never is subjected to excessive stresses over the entire range of operating conditions.

Description of the Prestressing Process

The prestressing process depends for its success on the existence of mortars which possess the ability to be cured in two stages. In the first stage, the mortars set hard enough to prevent the collapse of the brick lining, but are still deformable. The second stage cures them to a rigid, nondeformable state, but not before they have undergone a permanent swelling. It is this swelling that provides the prestressing.

The prestressing cure proceeds as follows: After the lining is installed and the first stage cure has occurred, the vessel is subjected to elevated temperature and pressure. The brickwork encounters tensile stresses due in part to the internal pressure and in part to the higher coefficient of thermal expansion of the steel jacket. The prestressing mortar swells, or stretches, to counteract the tension and to retain contact with the steel support. When the second stage of cure is complete, the mortar becomes rigid in its swollen state. After cooling, the brickwork is in compression and the steel jacket is in tension. When the vessel is subjected to elevated temperatures and pressures, compression recedes in the brickwork, but it never undergoes tensile stresses unless operating conditions more severe than the cure conditions are encountered.

Mathematical Analysis

The successful design of a prestressed brickwork lining must depend on a careful and somewhat elaborate analysis, as can be seen by the following factors which must be considered in the design:

(1) The prestressing mortars must have enough swelling capacity to accommodate, without disbonding, the tensile stresses imposed during the second stage of cure.

(2) The brickwork must be able to withstand the resulting higher compressive stresses that it will be subjected to during both operating and shutdown conditions.

(3) Prestressing will impose higher tensile stresses in the steel shell, which it must be designed to withstand.

(4) During operating conditions, the thermal gradient across the brick lining will cause bending stresses in the brickwork, which must not exceed material stress limits.

There are a great many variables in the design of a lining: choice of materials with differing physical characteristics, number and thickness of brickwork layers, width of brick and mortar joints, use of an impermeable membrane between brick and steel, and perhaps the use of an insulating jacket.

The mathematical techniques needed to analyze a design will be described next. Simplifying assumptions will be made so that the calculations can be carried out by hand. These assumptions will not produce significant errors in the analysis as long as the lining is thin relative to the radius of the vessel. Ideally, a computer program should be available, to avoid simplifying assumptions, to make it possible to analyze many designs quickly, and to eliminate the possibility of mistakes.

The analysis described is a refinement of that found in Reference 1.

METHODS OF ANALYSIS

Suppose now that the designer has developed a tentative design for the vessel, which includes the number of layers, material specifications for each layer, and the thickness of each layer, as well as the determination of the installation temperature, cure and operating conditions, and the maximum and minimum ambient temperatures that will be encountered both during operation and when idle. These operating and cure conditions to be specified are inside temperature and pressure, ambient temperature, and inside and outside heat transfer coefficient.

The analysis can be broken down into the following steps:

(1) Compute the composite physical properties of brick/mortar layers.

(2) Compute the cure temperatures and stresses in each layer.

(3) Check that the prestress swelling that will occur during cure does not exceed the capacity of the materials in the lining to swell. If swelling exceeds the maximum allowable, the lining will break up. Redesign and compute the analysis again.

(4) Calculate the temperatures and stresses in each layer for the extremes of operating and shutdown conditions. If stress limits are exceeded, redesign.

Composite Properties of Brick/Mortar Layers

Each layer composed of both brick and mortar must have its composite physical properties calculated as a combination of the properties of the brick and mortar components. The equations are: (See Table 7 for the key to the notation)

(1) $$E = (W_b + W_m)/(W_b/E_b + W_m/E_m)$$

(2) $$k = (W_b k_b + W_m k_m)/(W_b + W_m)$$

(3) $$\alpha = (W_b \alpha_b + W_m \alpha_m)/(W_b + W_m)$$

(4) $$q = W_m q_m/(W_b + W_m)$$

The composite value of Poisson's Ratio can be taken to be that of the brick.

Thermal Gradient Calculation

Temperature drops across each layer are calculated by standard heat transfer techniques. It is assumed that coefficients of thermal conductivity, as well as all physical properties, are temperature-independent (that is, uniform within the thermal range of operation).

The total thermal resistance, per axial foot in a cylindrical vessel, is given by

(5) $$R = 1/(2\pi r_i h_i) + 1/(2\pi r_o h_o) + \sum_{n=1}^{N} R_n$$

The first two terms are the thermal resistance of the inner and outer film, R_i and R_o respectively, and the third is the sum of the thermal resistance of each layer, computed by

$$R_n = [(\ln(r_n + d_n) - \ln(r_n)]/(2\pi k_n) \tag{6}$$

The temperature drop across the inner and outer film and across each layer is computed by the equations:

$$\delta T_i = (T_i - T_o)R_i/R \tag{7}$$

$$\delta T_o = (T_i - T_o)R_o/R \tag{8}$$

$$\delta T_n = (T_i - T_o)R_n/R \tag{9}$$

The temperature on the inside of the innermost layer is

$$T_o = T_i - \delta T_i \tag{10}$$

The temperature on the boundary between layers n and n+1 is given recursively by

$$T_n = T_{n-1} - \delta T_n \tag{11}$$

Finally, the average temperature in each layer is

$$\bar{T}_n = (T_{n-1} + T_n)/2 \tag{12}$$

Stress and Strain Calculations

The exact calculation of radial and circumferential stresses in each layer requires the solution of N+2 linear equations in N+2 unknowns, namely the N+1 radial displacements of the layer boundaries, and the longitudinal strain of the vessel. We simplify by assuming that the internal pressure is applied only to the steel shell, and that the other layers follow the expansion of the steel. We also assume a condition of plane stress; that is, no stress in the axial direction of the cylindrical vessel. We also consider the layer as being flat when layer stresses are being computed.

We now give formulas for computing the changes in stresses and strains in each layer when temperature-pressure conditions change. Suppose that, in changing to a new condition, the system undergoes a change Δp in internal pressure, and each layer encounters an average temperature change of $\Delta \bar{T}_n$. First we calculate the total change in strain, which can be expressed as the sum of strain changes due to temperature changes alone, and that due to pressure changes alone:

$$\Delta\epsilon = \Delta\epsilon_T + \Delta\epsilon_P \tag{13}$$

The two strain components are computed separately. First the pressure-induced component: It can be shown that a thin cylindrical shell of radius r, thickness d, and elastic modulus E, subject to inner pressure Δp, undergoes strain given by

$$\Delta\epsilon_P = \Delta pr/Ed \tag{14}$$

In a bonded lining, the effective elastic modulus is given by

$$E = \left(\sum_{n=1}^{N} E_n d_n\right)/d \tag{15}$$

where

$$d = \sum_{n=1}^{N} d_n \tag{16}$$

The value of r is taken to be that of the steel shell.

Next the temperature-induced strain is computed: The change in stress in each layer due to temperature change in that layer is determined by the difference between the total strain and the free thermal strain of the layer due to its average temperature change:

$$\Delta\bar{S}_{nT} = E_n(\Delta\epsilon_T - \alpha_n\Delta\bar{T}_n) \tag{17}$$

Equilibrium considerations imply that the average stress in the lining must be zero:

$$\sum_{n=1}^{N} \Delta\bar{S}_{nT} = 0 \tag{18}$$

Substituting in the above equation, the temperature-induced strain can be computed. Combining this with above calculations produces

$$\Delta\epsilon = \frac{\Delta p_r + \sum_{n=1}^{N} E_n d_n \alpha_n \Delta\bar{T}_n}{\sum_{n=1}^{N} E_n d_n} \tag{19}$$

The average change in stress in each layer is thus

$$\Delta\bar{S}_n = \alpha E_n(\Delta\epsilon - \alpha_n\Delta\bar{T}_n) \tag{20}$$

Finally, the temperature difference between the inner and outer surfaces of a layer produces an additional stress distribution, which varies from a maximum compression on the hot side to a maximum tension on the cold side. The maximum values are given by

$$S_{ng} = \tfrac{1}{2}\alpha_n E_n \delta T_n/(1 - \mu_n) \tag{21}$$

Thus the boundary stresses in layer n are given by

$$S_n = \Delta\bar{S}_n - S_{ng} \quad \text{(hot side)} \tag{22}$$

$$S_n = \Delta\bar{S}_n + S_{ng} \quad \text{(cold side)} \tag{23}$$

Stresses During and After Cure

The above formulae can be used for calculating the stresses induced in moving from installation conditions to all other conditions of cure, shutdown, and operation, as long as there is no prestress swelling. However, this procedure cannot be employed during a prestressing cure since the layers with swelling capacity are not perfectly elastic until the cure is complete.

The method for mathematically simulating the cure process is as follows: First, compute the layer stresses in moving from installation to cure conditions, assuming that all materials are perfectly elastic. The resulting stresses are examined, and the layers that appear to be under tensile stresses and which contain swellable materials are noted. These are the layers that will swell during cure to neutralize the tension.

The swelling is simulated mathematically by repeating the stress calculation with the elastic modulus for the noted layers temporarily considered as zero. This second calculation reflects the actual stresses in the layers at the end of the cure phase. For subsequent calculations the elastic moduli are restored to their original values.

Finally, it is necessary to check that the swelling that occurs in the noted layers does not exceed their maximum swelling capacity. The actual swelling is the total strain of the system minus the free thermal strain of the layer, thus the following condition must be satisfied:

$$q_n \geqslant \Delta\epsilon - \alpha_n \Delta\bar{T}_n \tag{24}$$

If any of these conditions fails, the lining must be redesigned by substituting different mortars, making the side joints thicker, using smaller-size bricks, and so on.

Stresses at Operating and Shutdown Conditions

Once the cure temperature distributions and stresses have been calculated, the stresses at various operating and shutdown conditions can be easily computed using formulas (13) to (24) by viewing the conditions simply as changes in temperatures and inner pressure.

Stresses must be checked for the severest possible conditions that the vessel will undergo. This may be at maximum operating temperatures and pressures at the extremes of idle and/or ambient temperatures.

If any layers are found to exceed temperature or stress maxima, the vessel's lining must be redesigned.

SAMPLE CALCULATION

Assume that specifications for the design of the vessel are as in Table 47-1, the lining consisting of three layers: one brick layer, a mortar bed layer, and the steel shell. This table also displays the calculated composite physical properties of the brick/mortar layers, computed according to equations (1) to (4).

Table 47-2 shows cure and operating conditions to be used in stress calculations.

Table 47-1: Proposed Vessel Design

Shape: Cylindrical
Number of layers: 3
Radius: 35.0 inches to inside of layer number 3

Layer		Description	Inner Radius	Thickness	CF TH EXP *E-6	Therm Cond
1	4.5"	Sample brick	33.5	1.25	2.56	8.06
	0.25"	Sample mortar	—	—	13.3	11.3
		Composite properties	—	—	3.13	8.23
2		Sample mortar	34.75	0.25	13.3	11.3
3		Sample steel	35.0	0.375	6.5	312.0

Layer		Elastic Modulus *E+6	Poisson Ratio	Maximum Compress	Maximum Tension	Swell Coeff *E-5
1	Brick	6.12	0.2	1600.0	400.0	0.0
	Mortar	0.426	0.17	2000.0	1800.0	700.0
	Composite	3.59	0.2	1600.0	400.0	36.8
2		0.426	0.17	2000.0	1800.0	700.0
3		29.0	0.3	13750.0	13750.0	0.0

Table 47-2: Installation, Cure, Operating, and Shutdown Conditions

Installation temperature	70°F
Cure conditions	
Inside temperature	240°F
Inside pressure	20 psi
Ambient temperature	80°F
Inner film conductance	300 Btu/ft^2-hr-°F
Outer film conductance	3 Btu/ft^2-hr-°F
Operating conditions	
Maximum operating temperature	220°F
Maximum pressure	15 psi
Ambient temperature range	0°-100°F

Table 47-3 shows cure calculations. The temperature gradient is computed by equations (5) to (12). Stresses are calculated using equations (13) to (24), first assuming all materials are perfectly elastic, and using changes in conditions from installation to cure (note the ambient temperature change of 10°, $\Delta p = 20$, installation stresses are zero). It is found that, among the layers containing swellable materials, only the brick/mortar layer is under tension, allowing the layer to swell. The last part of Table 47-3 shows the stresses calculated assuming a zero elastic modulus in those two layers.

Table 47-4 displays the stresses in the layers after the cure phase has cooled to the installation temperature. These stresses can be used as a base from which stress changes can be calculated for all other conditions.

Tables 47-5 and 47-6 show the temperatures and stresses, again calculated by equations (5) to (24), for various operating and shutdown conditions. It is seen that the stresses under all conditions are within specified limits.

Table 47-3: Cure Temperatures and Stresses

	Layer Number		
	1	2	3
Assuming No Swelling			
Hot side temperature	238.9	190.3	183.3
Cold side temperature	190.3	183.3	183.0
Hot side stress	565	-386	-962
Percent of maximum	141	19	7
Cold side stress	1247	-338	-862
Percent of maximum	312	17	6
Assuming Swelling (Layer 1 Elastic Modulus = 0)			
Hot side temperature	238.9	190.3	183.3
Cold side temperature	190.3	183.3	183.0
Hot side stress	-341	-342	2028
Percent of maximum	21	17	15
Cold side stress	341	-294	2129
Percent of maximum	85	15	15
Percent maximum swell	96	—	—

Table 47-4: Stresses After Cure—Cooling to 70°F

	Layer Number		
	1	2	3
Average stress	-906	44	2991
Percent of maximum	57	2	22

Table 47-5: Stresses Under Extreme Operating Conditions

Maximum Summer Operating Conditions

Inside temperature: 220°F
Outside temperature: 100°F
Pressure: 15 psi
Inner film conductance: 300.0 Btu/ft^2/°F
Surface conductance: 3.0 Btu/ft^2/°F

	Layer Number		
	1	2	3
Hot side temperature	219.2	182.7	177.5
Cold side temperature	182.7	177.5	177.2
Hot side stress	-285	-319	1660
Percent of maximum	18	16	12
Cold side stress	227	-284	1736
Percent of maximum	57	14	13

Maximum Winter Operating Conditions

Inside temperature: 220°F
Outside temperature: 0°F
Pressure: 15 psi
Inner film conductance: 300.0 Btu/ft^2/°F
Surface conductance: 3.0 Btu/ft^2/°F

	Layer Number		
	1	2	3
Hot side temperature	218.5	151.6	142.1
Cold side temperature	151.6	142.1	141.6
Hot side stress	-966	-222	3113
Percent of maximum	60	11	23
Cold side stress	-27	-157	3251
Percent of maximum	2	8	24

Table 47-6: Stresses Under Shutdown at Ambient Extremes

	Layer Number		
	1	2	3
Winter Shutdown—Ambient Temperature: 0°F			
Average stress	-1520	275	4884
Percent of maximum	95	15	36
Summer Shutdown—Ambient Temperature: 100°F			
Average Stress	-643	-55	2179
Percent of maximum	40	3	16

Table 47-7

Symbols and Notation

E	Elastic modulus, lb/sq in
k	Thermal conductivity, Btu/ft^2-hr-°F
q	Swelling coefficient, dimensionless
α	Coefficient of thermal expansion, in/in-°F
μ	Poisson's Ratio, dimensionless
h	Film heat transfer coefficient, Btu/ft^2-hr-°F
N	Number of layers in the lining
R	Thermal resistance, per axial foot
T	Temperature
T_n	Temperature at outside of layer n
$\overline{T}$	Average temperature in a lining
δT	Temperature drop across a layer
Δ	Change in a parameter when conditions change
S	Circumferential stress
ϵ	Circumferential strain
d	Thickness, inches
W	Width of brick or mortar, inches

Subscripts

a	Ambient, or outer surface
b	Brick
m	Mortar
i	Inner surface of lining
n	nth layer
r	Radius
g	Stress due to temperature difference across layer

SUMMARY AND CONCLUSIONS

The calculations described in this chapter apply only to cylindrical vessels whose lining is thin relative to the radius of the vessel. Thick-walled vessels, and vessels of other shapes such as rectangular or spherical, will require considerably more complex mathematical analysis which is beyond the scope of this handbook. In such cases, exact formulae are difficult or impossible to obtain, and the designer must resort to computer programs for performing the complex calculations.

This chapter also does not consider the analysis of stresses around piping, connections, supports, attachments, and so on. While the experienced engineer can design a vessel to prevent failure at these locations, accurate analysis requires elaborate techniques such as the Finite Element Method. This method has been applied with great success to analyze complex vessels such as nuclear reaction vessels. The reader should consult appropriate references if he wishes to pursue this area. References 2 and 3 are basic textbooks in the field of finite element analysis.

REFERENCES

1. Honigsberg, C.A. and Eschenbrenner, G.P., Prestressed non-metallic vessel linings, *Chem. Eng. Prog.,* Vol 58, September, 1962, pp 81–84, and Vol 58, October, 1962, pp 97–101.
2. Desai, C.S. and Abel, J.F., *Introduction to the Finite Element Method,* Van Nostrand Reinhold, New York, 1972.
3. Tong, P. and Rossettos, J.N., *Finite-Element Method,* The MIT Press, Cambridge, 1977.

Editor's Note: In determining the insulation (number and thickness of masonry layers) required to keep the surface temperature of the membrane at acceptable levels, the designer may make use of a simplified calculation which is sufficiently accurate for this purpose. The procedure is detailed in *Chemically Resistant Masonry,* by Walter Lee Sheppard, Jr. (2nd Ed., 1982, Marcel Dekker) pages 112–113.

It may be summarized thus:

(1) Insulation factor of system,

$$R = \frac{\text{thickness of layer}}{\text{K factor of layer}} + \frac{\text{thickness}}{\text{K factor}} + \frac{\text{thickness}}{\text{K factor}} \quad \ldots \quad (R_I)$$

(2) Thermal drop of system,

$$DT = \text{operating temp.} - \text{ambient temp. (usually } 70^{\circ}\text{F)}$$

(3) Thermal drop for each inch (or other unit) of lining then is,

$$DT_n = \frac{DT}{R}; \text{ while for each layer } DT_I = R_I \times DT_n$$

Applying these simplified formulae the designer can quickly determine if his design provides sufficient insulation (thickness) or if he requires another layer. The cited reference provides examples of these calculations.